Practice for Units 6 & 7 Name Key

Station 1 – Even and Odd Functions

Read and answer the questions on the back.

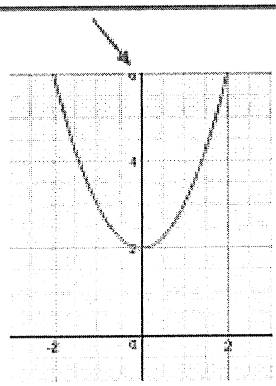
A **Function** can be classified as ***Even, Odd*** or ***Neither***. This classification can be determined *graphically* or *algebraically*.

Graphical Interpretation -

Even Functions:

Have a graph that is symmetric with respect to the **Y-Axis.**

Y-Axis – acts like a mirror

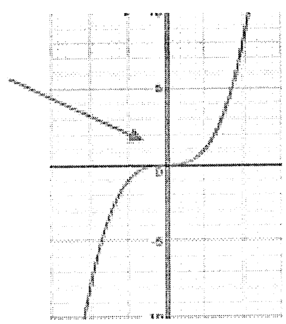


Odd Functions:

Have a graph that is symmetric with respect to the **Origin.**

Origin – If you spin the picture upside down about the Origin, the graph looks the same!

Origin



Algebraic Test – Substitute $(-x)$ in for x everywhere in the function and analyze the results of $f(-x)$, by comparing it to the original function $f(x)$.

Even Function: $y = f(x)$ is ***Even*** when, for each x in the domain of $f(x)$, $f(-x) = f(x)$

Odd Function: $y = f(x)$ is ***Odd*** when, for each x in the domain of $f(x)$, $f(-x) = -f(x)$

Examples:

a. $f(x) = x^2 + 4$

$f(-x) = (-x)^2 + 4$

$f(-x) = x^2 + 4$

$f(-x) = f(x)$

Even Function!

b. $f(x) = x^3 - 2x$

$f(-x) = (-x)^3 - 2(-x)$

$f(-x) = -x^3 + 2x$

$f(-x) = -(x^3 - 2x) = -f(x)$

Odd Function!

c. $f(x) = x^2 - 3x + 4$

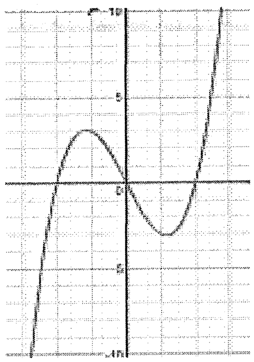
$f(x) = (-x)^2 - 3(-x) + 4$

$f(-x) = x^2 + 3x + 4$

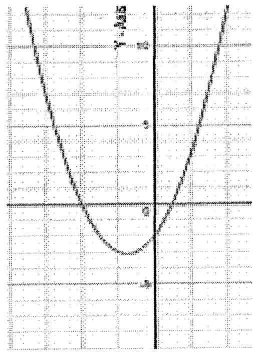
$f(-x) \neq f(x) \neq -f(x)$

Neither!

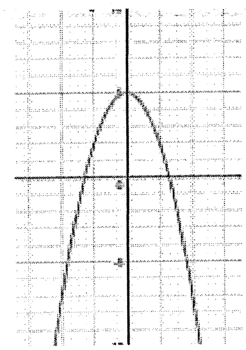
A. Graphically determine whether the following functions are Even, Odd, or Neither

1. 

odd
symmetric
about the
origin

2. 

neither

3. 

even
symmetric
about the
y-axis

B. Algebraically determine whether the following functions are Even, Odd, or Neither

1. $f(x) = x^3 - x^2 + 4x + 2$

$f(-x) = (-x)^3 - (-x)^2 + 4(-x) + 2$ neither

$f(-x) = -x^3 - x^2 - 4x + 2$

2. $f(x) = -x^2 + 10$

$f(-x) = -(-x)^2 + 10$ even

$= -x^2 + 10$ $f(x) = f(-x)$

3. $f(x) = x^3 + 4x$

$f(-x) = (-x)^3 + 4(-x)$ odd

$-x^3 - 4x$ $-f(x) = f(-x)$

4. $f(x) = -x^3 + 5x - 2$

$f(-x) = -(-x)^3 + 5(-x) - 2$ neither

$x^3 - 5x - 2$

C. If $f(x)$ is an odd, one-to-one function with $f(5) = -2$ then which point *must* lie on the graph of its inverse $f^{-1}(x)$?

$(5,-2)$ inverse = swap

$(-5,2) \leftarrow$ odd x & y

(1) $(5,-2)$ (2) $(2,-5)$ (3) $(-5,2)$ (4) $(2,5)$

$(-2,5)$

$(2,-5)$

Station 2 – Discriminant

The discriminant is a small part of the quadratic formula.

$$b^2 - 4ac$$

Find the discriminant to determine the number of x-intercepts and the nature of the roots.

1. $2x^2 - 3x + 2 = 0$

$b^2 - 4ac$

$(-3)^2 - 4(2)(2) = -7 \Rightarrow$ imaginary, unequal

no x-int

2. $3x + 7 = 5x^2 - 4$

$5x^2 - 3x - 11 = 0$

$(-3)^2 - 4(5)(-11) = 229 \Rightarrow$ real, irrational, unequal

2 x-int

3. $9x^2 + 24x + 16 = 0$

$(-24)^2 - 4(9)(16) = 0 \Rightarrow$ real, rational, equal

1 x-int

4. $x^2 - 7x + 6 = 0$

$(-7)^2 - 4(1)(6) = 25 \Rightarrow$ real, rational, unequal

2 x-int

Challenge:

5. Find all the values of c such that $2x^2 - 6x + c = 0$ has unequal, imaginary roots.

$b^2 - 4ac < 0$

$(-6)^2 - 4(2)(c) < 0$

$36 - 8c < 0$

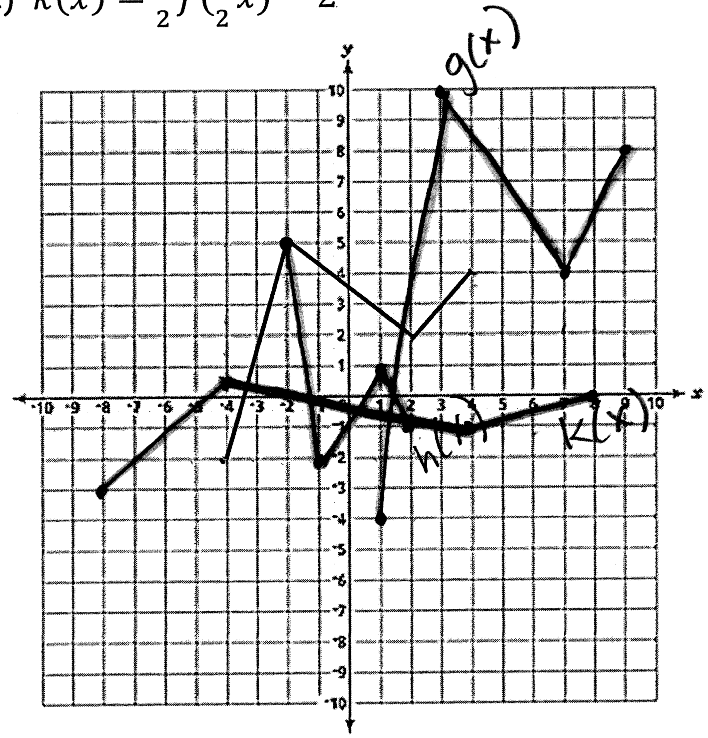
$-8c < -36$

$c > 4.5$

Station 3 - Transformations

1. Given the graph of $f(x)$, sketch and the label the graph of the following functions using a different color for each.

   a) $g(x) = 2f(x-5)$
   b) $h(x) = -f(2x)+3$
   c) $k(x) = \frac{1}{2}f(\frac{1}{2}x) - 2$



Transformations of the graphs of functions	
$f(x)+c$	shift $f(x)$ up c units
$f(x)-c$	shift $f(x)$ down c units
$f(x+c)$	shift $f(x)$ left c units
$f(x-c)$	shift $f(x)$ right c units
$f(-x)$	reflect $f(x)$ about the y-axis
$-f(x)$	reflect $f(x)$ about the x-axis
$cf(x)$	When $0<c<1$ – vertical shrinking of $f(x)$ When $c>1$ – vertical stretching of $f(x)$ Multiply the y values by c
$f(cx)$	When $0<c<1$ – horizontal stretching of $f(x)$ When $c>1$ – horizontal shrinking of $f(x)$ Divide the x values by c

2. If $f(x)=x+10$ and $g(x)=f(2x)$ then $g(-3)=$

   (1) 7 (3) -30

   (2) 2 (4) 4

$g(-3) = f(2\cdot -3)$
$= f(-6)$
$f(-6) = -6+10$
$= 4$

3. Suppose the point $(6,-1)$ is a point of the graph of $f(x)$. For each of the following, state the coordinate of the point after the transformation.

   a) $y = f(3x)$ (2, -1)

   b) $y = f(x+2)$ (4, -1)

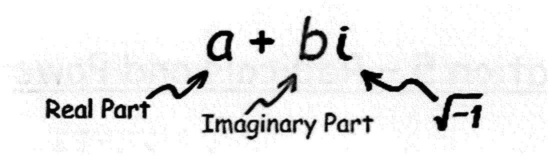
   c) $y = f(x)+5$ (6, 4)

   d) $y = \frac{1}{2}f(x) - 3$ (6, -3.5)

   e) $y = -4f(x-1)+2$ (7, 6)

   f) $y = f\left(-\frac{1}{3}x\right) - 7$ (-18, -8)

Station 4 – Solving Equations



1. Solve the equation. Leave answers in simplest a + bi form.

$$8x^2 + 4x + 5 = 0$$

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(8)(5)}}{2(8)}$$

$$= \frac{-4 \pm \sqrt{-144}}{16} \quad = \frac{-4 \pm 12i}{16} \quad = -\frac{1}{4} \pm \frac{3}{4}i$$

2. Solve the equation. Check for extraneous solutions.

$$\sqrt{x+2} - 3 = 2x$$

$$\sqrt{x+2}^{\,2} = (2x+3)^2$$

$$x + 2 = 4x^2 + 12x + 9$$

$$4x^2 + 11x + 7 = 0$$

$$x = \frac{-11 \pm \sqrt{(11)^2 - 4(4)(7)}}{2(4)}$$

$$x = \frac{-11 \pm \sqrt{9}}{8}$$

$$x = \frac{-11 \pm 3}{8}$$

$$x = -1, \cancel{-\frac{7}{4}}$$

check

$$\sqrt{-1+2} - 3 = 2(-1)$$

$$1 - 3 = -2 \checkmark$$

$$\sqrt{-\frac{7}{4}+2} - 3 = 2\left(-\frac{7}{4}\right)$$

$$-2.5 \neq -3.5$$

3. Solve the equation.

$$\frac{3(x+3)^{\frac{3}{4}}}{3} = \frac{81}{3}$$

$$\left((x+3)^{\frac{3}{4}}\right)^{4/3} = (27)^{4/3}$$

$$x + 3 = 81$$

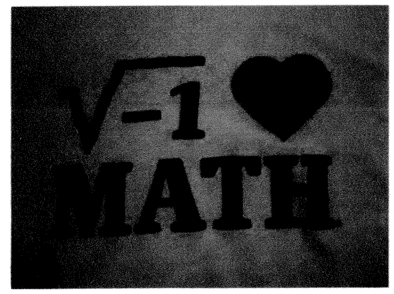
$$x = 78$$

check

$$3(78+3)^{\frac{3}{4}} = 81$$

$$3(81)^{3/4} = 81 \checkmark$$

Station 5 – Radicals and Powers of i



1) What is the product of $\sqrt[3]{4a^2b^4}$ and $\sqrt[3]{16a^3b^2}$?

a. $4ab^2\sqrt[3]{a^2}$

b. $4a^2b^3\sqrt[3]{a}$

c. $8ab^2\sqrt[3]{a^2}$

d. $8a^2b^3\sqrt[3]{a}$

$\sqrt[3]{64a^5b^6}$

$4ab^2\sqrt[3]{a^2}$

2) Given i is the imaginary unit, $(2 - yi)^2$ in simplest form is

a. $y^2 - 4yi + 4$

b. $-y^2 - 4yi + 4$

c. $-y^2 + 4$

d. $y^2 + 4$

$4 - 4yi + y^2 \underbrace{i^2}_{(-1)}$

$-y^2 - 4yi + 4$

3) The expression $3\sqrt{-18} + 5\sqrt{-12}$ is equivalent to

a. $9\sqrt{2}i + 10\sqrt{3}i$

b. $6\sqrt{2}i + 7\sqrt{3}i$

c. $19\sqrt{5}i$

d. $-90\sqrt{6}$

$3(3\sqrt{2}i) + 5(2\sqrt{3})i$

$9\sqrt{2}i + 10\sqrt{3}i$

4) What is the sum of $5 - 3i$ and the conjugate of $3 + 2i$?

a. $2 + 5i$

b. $2 - 5i$

c. $8 + 5i$

d. $8 - 5i$

$(5 - 3i) + (3 - 2i)$

$8 - 5i$

5) Simplify. Rationalize the denominator.

$$\frac{2-\sqrt{3}}{4+\sqrt{3}} \cdot \frac{4-\sqrt{3}}{4-\sqrt{3}} = \frac{8 - 2\sqrt{3} - 4\sqrt{3} + 3}{16 - 3}$$

$$= \frac{11 - 6\sqrt{3}}{13} = \frac{11}{13} - \frac{6\sqrt{3}}{13}$$